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String creation, D-branes and effective field theory

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Abstract: This paper addresses several unsettled issues associated with string creation in systems of orthogonal Dp-D$(8-p)$ branes. The interaction between the branes can be understood either from the closed string or open string picture. In the closed string picture it has been noted that the DBI action fails to capture an extra RR exchange between the branes. We demonstrate how this problem *persists* upon lifting to M-theory. These D-brane systems are analysed in the closed string picture by using gauge-fixed boundary states in a non-standard lightcone gauge, in which RR exchange can be analysed precisely. The missing piece in the DBI action also manifests itself in the open string picture as a mismatch between the Coleman-Weinberg potential obtained from the effective field theory and the corresponding open string calculation. We show that this difference can be reconciled by taking into account the superghosts in the (0+1) effective theory of the chiral fermion, that arises from gauge fixing the spontaneously broken world-line local supersymmetries.

Keywords: Anomalies in Field and String Theories, D-branes, Supersymmetric Effective Theories.

Contents

1. Introduction

It was first observed in [1] that when a D5 brane aligned along $x^i, i \in \{0,1,2,3,4,5\}$ crosses an NS5 brane aligned along $x^j, j \in \{0,1,2,6,7,8\}$ in the x^9 direction, a D3 brane aligned along $x^k, k \in \{0,1,2,9\}$ is created. This is the Hanany-Witten effect [1]. Examining the NS 3-form flux induced by the NS5 brane on the D5 world-volume leads to the conclusion that a D3 brane has to be created, by virtue of Gauss's theorem, given that the induced flux jumps by one unit as the brane crosses each other.

By a string of T and S dualities, the Hanany-Witten configuration can be mapped to a system of orthogonal Dp-D$(8-p)$ branes, either in type IIA or IIB, oriented so that there is a single spatial direction transverse to the branes [2]. The creation of D3 branes translates into the creation of F1 strings connecting the two branes as they cross. This phenomenon again directly follows from Gauss's theorem. In fact, instead of considering charge conservation in brane world-volume as in [1], we could equally look at the Gauss's equation of the NS 3-form flux to which fundamental strings are coupled [3]. Consider for example a D0 and a D8 brane in IIA. Gauss's equation for the NS 3-form is given by

$$d \star (e^{-2\phi} dB_2) = 2m \star (dA_1 + mB_2), \tag{1.1}$$

where B_2 is the NS 2-form potential, A_1 is the RR 1-form potential under which D0 is charged, and m is the cosmological constant, dual to F_{10} which couples to D8. Suppose the D8 brane wraps around an 8-sphere. Integrating both sides of the equation along the 8-sphere, the l.h.s is identically zero. However, the r.h.s is not if the D0 brane is enclosed by the D8 due to the contribution of $2m \star dA_1$. We thus conclude that as the D0 crosses the D8 from outside the 8-sphere, a string which sources the NS 3-form, has to be created, giving rise to an extra term $\sim (2\pi\alpha')^{-1}\delta^{(8)}(x)$ on the r.h.s., which cancels the D0 contribution. Taking the limit that the radius of the eight-sphere approaches infinity, we recover the original story of string creation as the D0 crosses D8. This argument can be readily generalised to other Dp-D$(8-p)$ systems, in which case Gauss's equation for the NS 3-form is given by (in type IIA for concreteness)

$$d \star (e^{-2\phi} dB_2) = -\frac{1}{2} F_4 \wedge F_4 + F_2 \wedge \star F_4 - \star A_1 \wedge H_3 \wedge F_2, \tag{1.2}$$

where the first two terms on the l.h.s. account for string creation in all the possible Dp-D$(8-p)$ systems in IIA, including D4-D4 and D2-D6 respectively. Clearly from T-duality similar equations can be written down for IIB.

This phenomenon, however, raises questions. By open-closed duality, the interaction between the branes can be analysed either as a closed string tree level exchange or an open string one-loop potential. From the closed string picture, a boundary state calculation gives two pieces, from NSNS and RR exchange respectively, with equal magnitude. It is given by the product of half of the tension of a string and separation. However, the RR exchange has a sign ambiguity. Starting with one convention the sign flips when the two branes cross each other. Therefore beginning with a vanishing potential between the branes, a linear potential is created when the branes cross. By virtue of supersymmetry, which asserts that the force should vanish, we have to conclude that this nontrivial potential is cancelled by a string that is created as the branes cross. This is consistent with the above analysis via Gauss's theorem.

Yet this crucial RR exchange is unexpected since naively the two branes couple to different RR potentials. The DBI action fails to capture this extra interaction and thus a probe Dp brane in the curved background generated by the D$(8-p)$ brane experiences a net force due to the gravitons and dilaton. This is reviewed in section 2. In the case of a probe D2 brane in a D6 background the system can be lifted to M-theory. The D2 brane becomes and M2 brane and the D6 becomes a Taub-Nut background. The extra duality between the RR fields then implies a duality between the three form potential and the metric. It is then found that as in the type II limit, the naive membrane action also *misses* the extra contribution.

A careful analysis in string theory of the unexpected RR exchange [5] using the boundary state formalism suggests that it can be ascribed to an extra duality relationship between the time-like modes which are otherwise decoupled from the string S-matrix calculation in the absence of boundary sources. The non-zero overlap between these time-like modes arises from a cancellation between the zeroes from the trace of Gamma matrices and divergences from the ghost sector, upon application of a regularisation procedure. As we will show, the duality relationship so obtained is more generic than is described in [5] and can be shown to exist between many other components of the potential. We will also see that the duality relation proposed is neither Lorentz invariant nor gauge invariant. These problems will be reviewed in section 3. So the question arises as to how a Lorentz invariant formalism could have given rise to such a duality relationship. The main feature of our analysis is the choice of a particular gauge, the closed string generalisation of the Arvis gauge [4, 6], which is a variant on the light-cone gauge that avoids the problem of divergent ghost contributions. We then attack the problem using boundary methods without the need to confront divergences arising in the ghost sector. As we shall see, in section 4, the same results as in [5] are produced. However, this formulation makes it clear that the duality relations are only an artifact of an explicit gauge choice. The unexpected RR exchange, is only possible when both branes are present, where some of the ten-dimensional Lorentz symmetry is broken. Neither the DBI action nor the supergravity solutions of parallel D-branes, are affected by these extra duality relations.

The problem can be analysed entirely in the open string language. In the open string description the interactions between the branes manifest themselves as a 1-loop Coleman-Weinberg effective potential after integrating out the open string ground states that connect the two branes. The open string calculation matches exactly that of the closed string calculation. The NSNS contribution comes, in the open string language, from the sum of two different sectors, namely

$$\frac{1}{2}\mathrm{tr}_{\mathrm{NS}} q^{L_0 - a_{\mathrm{NS}}} + \frac{1}{2}\mathrm{tr}_{\mathrm{R}} q^{L_0}. \tag{1.3}$$

The RR contribution, comes from

$$\frac{1}{2}\mathrm{tr}_{\mathrm{R}}\left((-1)^F q^{L_0}\right). \tag{1.4}$$

An effective theory calculation however, gives only the contribution from (1.3) and misses those from (1.4) [7]. For consistency and supersymmetry, the extra contribution from (1.4) is put in by hand in the effective theory and it is known as the *bare term*. We have seen that in either the closed string or open string description, the contribution (1.4) plays a crucial yet mysterious role. It has also been referred to as a *half-string*, [3, 8] as an explanation of its stringy origin, since the term gives rise to a potential of half that of a string. The effective theory should provide a microscopic explanation of the term

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since it should be possible to map every detail in the effective theory to the open string calculation. We show that it is a quantum effect whose origin could be traced to $\beta\gamma$ ghosts in the effective field theory description. The discussion is presented in section 5.

2. Dp brane probes in D$(8-p)$ backgrounds

The general form of a Dp brane supergravity solution is

$$\begin{aligned} ds^2 &= H_p^{-1/2} dx_{p+1}^2 + H_p^{1/2} dx_{9-p}^2, \\ e^{2\phi} &= H_p^{\frac{3-p}{2}}, \\ H &= 1 + \frac{h_p}{r^{7-p}}, \end{aligned} \tag{2.1}$$

for some constants h_p. Substituting the above metric into the DBI action of a probe Dq brane with eight relatively transverse dimensions and switching off all kinetic terms we readily obtain the potential term given by

$$T_q \int d^{q+1}\sigma e^{-\phi} \sqrt{H_p^{\frac{-1+n-m}{2}}} = T_q \int d^{q+1}\sigma H_p^{\frac{p-3-m+n-1}{4}}, \tag{2.2}$$

where there are m shared directions between the Dp and Dq brane and $p-m+n=8$ relatively transverse directions. This gives a potential proportional to

$$V(r) = T_q \int d^{q+1}\sigma H_p, \tag{2.3}$$

which is clearly non-vanishing. This apparent contradiction with intuition about BPS configurations was observed long ago [9]. Nevertheless, general solutions of probe Dp branes in curved D$(8-p)$ backgrounds have been found [10−13] and are shown to exhibit the same behaviour as the D2 brane in D6 background, which we will discuss in further detail in the next subsection. These solutions are found by first finding a solution corresponding to the Dq probe wrapping the $(9-p)$ sphere transverse to the background Dp brane, with fields depending only on the polar angle θ. In other words, we solve for the field r, the radial coordinate of the transverse sphere, and the worldvolume electric field E as a function of θ only. Then the brane is decompactified by changing to coordinates

$$z = -r\cos\theta, \qquad \rho = r\sin\theta. \tag{2.4}$$

One aligns the probe brane along ρ instead and obtain z and E_ρ as a function of ρ. Starting from the expression for $\frac{r'}{r}$ [12] and differentiating (2.4), we finally obtain

$$\frac{dz}{d\rho} = \frac{D_\rho}{\rho^3 H_p}, \tag{2.5}$$

where the displacement field $D_\rho = \frac{\partial L}{\partial E_\rho}$. The probe action can also be simplified to

$$T_q \int dt d\rho H_p \rho^{q-1} \sqrt{1 - E_\rho^2 + \left(\frac{dz}{d\rho}\right)^2}. \tag{2.6}$$

The important lesson from the exercise is that we obtain finally

$$E_\rho = \frac{dz}{d\rho}, \tag{2.7}$$

which satisfies the BPS condition required by Kappa symmetry [16],

$$\frac{1}{\sqrt{1+z'^2-E^2}}\Gamma_{11}^{p-2/2}[\Gamma_{0\rho\phi_1\ldots} + (\Gamma_{0z}z' + \Gamma_{11}E)\Gamma_{\phi_1\ldots}]\epsilon = \epsilon, \tag{2.8}$$

as also observed in [13] for the case of a probe D5 in D3 background. There are only two independent projections on the killing spinors, from the background brane and the first term of the above equation and so the system should be quarter BPS. While the solution is shown to preserve supersymmetry, on closer inspection it turns out that there is a net force equal to half of the string tension pulling on the probe brane. The solution is static since the brane has infinite extent. This is a general feature of all such Dp probe solutions in IIA and IIB D$(8-p)$ supergravity backgrounds [10–13]. We will present an explicit calculation of this potential experienced by the D2 probe in the next subsection in the context of M-theory.

2.1 Example in M-theory

A probe D2 brane in a D6 background can be lifted to M-theory. In fact, the first example of an explicit solution of string creation from a probe brane perspective was given in [14] in the M-theory context. A probe D2 brane is placed in the curved background generated by the D6 brane. The system is lifted to M-theory where the D2 becomes an M2 and the D6, a KK monopole. The curved metric generated by a KK monopole is equivalent to a Taub-Nut space, which is a complex manifold and determines a special complex structure. The metric of a KK monopole located at the origin can be written, in the Einstein frame, as [15]

$$ds^2 = dx_7^2 + V dv d\bar{v} + \frac{1}{V}\left(\frac{dy}{y} - f dv\right)\overline{\left(\frac{dy}{y} - f dv\right)}, \tag{2.9}$$

where dx_7^2 denotes the Minkowski metric along the 6+1 dimensional world-volume of the KK monopole and

$$\begin{aligned} v &= \frac{(x^7+ix^8)}{R}, & h &= \frac{x^9}{R}, \\ \sigma &= \frac{x^{10}}{R}, & y &= e^{-(b+i\sigma)}\left(-b - + \sqrt{(b)^2+|v|^2}\right)^{\frac{1}{2}}, \\ V &= 1 + \frac{1}{2\sqrt{|v|^2+b^2}}, & f &= \frac{b+\sqrt{|v|^2+b^2}}{2|v|\sqrt{|v|^2+b^2}}. \end{aligned} \tag{2.10}$$

This background is 1/2 BPS. Any solution of the membrane action respecting the background complex structure would automatically satisfy a BPS condition and preserves 1/2 of the background supersymmetries. A supersymmetric M2 brane orthogonal to the KK monopole can thus be described by a first order holomorphic curve in v and y. Higher order curves would describe multiple M2 branes since they can be factorised as products of first order curves. There are two such distinct holomorphic curves, namely [14]

$$y = e^{-b}, \qquad y = e^{-b} v. \tag{2.11}$$

Considering their radial profiles it turns out they are mirror images of each other. The IIA limit is taken by sending all length scales including $|x^9|, |v|$ and b to infinity relative to x^{10}, while keeping the ratios among them fixed. The parameter b then describes the asymptotic x^9 position of the D2 brane. Suppose we consider the first curve in (2.11) with $b < 0$. The D2 brane is located near $x^9 \sim b$ for all v. As b is increased the brane is increasingly curved and as b passes through $b = 0$, the D2 brane is hooked up at the origin and while it still asymptotes to $x^9 \sim b$ at $v \sim \infty$, near $v \sim 0$, $x^9 \sim 0$. The shape of the curved probe becomes exactly that of a string stretching between the D2 brane at $x^9 \sim b$ and the D6 brane at $x^9 = 0$.

To show explicitly that the stretched part is indeed a string, we can calculate the tension of the D2 near $v \sim 0$ for $b > 0$. The energy of the system is simply given by the tension multiplied by the area of the brane. This is given by

$$E = T_2 \int d^2\zeta \sqrt{\det G_{ab}}, \qquad G_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}, \tag{2.12}$$

where T_2 is the brane tension and G_{ab} is the induced metric on the brane. The complex structure of the background simplifies the problem greatly. The expression of the energy is reduced to

$$E = \frac{T_2}{2} \int dz d\bar{z} g_{\mu\bar{\nu}} \partial_z X^\mu \partial_{\bar{z}} X^{\bar{\nu}}, \tag{2.13}$$

where $z = \zeta_1 + i\zeta_2$. For concreteness consider the second curve $w = \exp(-b)v$.[1] Since we are aligning the brane asymptotically along x^7, x^8 we choose the static gauge

$$z = Rv. \tag{2.14}$$

As a result we have

$$E = T_2 \int \frac{dv d\bar{v}}{2} \left(V + \frac{f^2}{V} \right). \tag{2.15}$$

To take the IIA limit and zoom into the region occupied by the extended string, we take

$$R = 1, \qquad x_9, b, |v| \sim +\infty, \qquad x_9 \ll b, \qquad \frac{|v|}{x_9} \ll 1. \tag{2.16}$$

In this limit

$$\begin{aligned}
|v|^2 &\sim e^{2(x_9 - b)} 2x_9, \\
|v|^2 + x_9^2 &\sim 2e^{(-2b)} x_9 + x_9^2 \sim x_9^2, \\
V &\sim 1 + \frac{1}{2x_9} \sim 1, \\
f^2 &\sim \frac{1}{|v|^2}.
\end{aligned} \tag{2.17}$$

[1] The second curve is wrapping the 11th direction while the first curve is not. Therefore in the rest of the section we shall restrict our attention to the second curve.

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Changing variables from z to x_9, we have

$$E = 2\pi T_2 \int e^{-2b(1-x_9/b)} dx_9 (1 + 2x_9 + 2e^{(2x_9+b)}) \left\{ 1 + \frac{1}{2x_9} + \frac{1}{|v|^2 \left(1 + \frac{1}{2x_9}\right)} \right\} = 2\pi T_2 \int dx_9. \tag{2.18}$$

Given that

$$T_{M2} = \frac{1}{2\pi l_s^2 2\pi g_s}, \tag{2.19}$$

and that $g_s = 1$ in our units, the energy in this region of the brane is given finally by

$$\int \frac{1}{2\pi l_s^2} dx_9. \tag{2.20}$$

The result agrees with the fact that a fundamental string connects the D2 to the D6. This argument readily generalises to the case where there are N D6 branes, in which case the modified metric would lead to N strings connecting the branes. The calculation of the brane energy can be done exactly. Observing the following relations after using $w = \exp(-bv)$

$$\begin{aligned} |v|^2 &= e^{2(x_9-b)} \left(2x_9 + e^{2(x_9-b)}\right) \\ |r| &= \sqrt{x_9^2 + |v|^2} = x_9 + e^{2(x_9-b)}, \qquad\qquad f = \frac{|v| e^{-2(x_9-b)}}{2|r|}, \end{aligned} \tag{2.21}$$

the integral

$$E = 2\pi T_2 \int dx_9 e^{2(x_9-b)} \left(1 + 2x_9 + 2e^{2(x_9-b)}\right) \left\{ 1 + \frac{1}{2|r|} + \frac{|v|^2 e^{-4(x_9-b)}}{2|r|(1+2|r|)} \right\}. \tag{2.22}$$

This can be simplified to

$$\begin{aligned} E &= 2\pi T_2 \left\{ \int dx_9 \left[\frac{e^{2(x_9-b)}}{2|r|} + e^{2(x_9-b)} + \frac{|v|^2 e^{-2(x_9-b)}}{2|r|} \right] + \int dx_9 e^{2(x_9-b)} (1 + 2|r|) \right\} \\ &= 2\pi T_2 \left\{ \int dx_9 \left[\frac{e^{2(x_9-b)} + x_9 + |r|}{2|r|} + e^{2(x_9-b)}) \right] + \int dx_9 e^{2(x_9-b)} (1 + 2|r|) \right\} \\ &= 2\pi T_2 \left\{ \int dx_9 (1 + 2e^{2(x_9-b)}) + \int |v| d|v| \right\}. \end{aligned} \tag{2.23}$$

The integral is divergent due to the infinite extent of the brane. The second bracket corresponds to the area of the membrane in a flat background. Since it is independent of the parameter b we will ignore it in later expressions. The curve is only defined between $0 < x_9 < b$. Therefore,

$$E = 2\pi T_2 \left\{ \frac{2x_9 + e^{2(x_9-b)}}{2} \right\}_0^\infty = 2\pi T_2 \left\{ \frac{|v|^2}{2} \left[1 + e^{-2(x_9-b)} \right] \right\}_0^\infty. \tag{2.24}$$

To extract the net force on the brane from the infinite energy due to the infinite extent of the brane, we differentiate the expression with respect to b while holding v fixed. This gives

$$F \equiv \left.\frac{dE}{db}\right|_v = 2\pi T_2 \left[\frac{|v|^2}{2} \left.\frac{de^{-2(x_9-b)}}{db}\right|_v \right]_0^\infty. \tag{2.25}$$

Note that

$$\left.\frac{de^{-2(x_9-b)}}{db}\right|_v = -2e^{-2(x_9-b)}\left(\left.\frac{dx_9}{db}\right| - 1\right)$$
$$= -2e^{-2(x_9-b)}\left(\frac{2e^{2x_9}+2x_9e^{2b}}{2e^{2x_9}+e^{2b}(1+2x_9)} - 1\right)$$
$$= \frac{2e^{-2(x_9-b)}e^{2b}}{2e^{2x_9}+e^{2b}(1+2x_9)}. \tag{2.26}$$

As a result,

$$F = 2\pi T_2\left[\frac{|v|^2}{2}\left.\frac{de^{-2(x_9-b)}}{db}\right|_v\right]_0^\infty = 2\pi T_2\left[\frac{e^{2b}(2x_9+e^{2(x_9-b)})}{2e^{2x_9}+e^{2b}(1+2x_9)}\right]_0^\infty. \tag{2.27}$$

In the limit $x_9 \to \infty, x_9 \gg b$,

$$\left[\frac{e^{2b}(2x_9+e^{2(x_9-b)})}{2e^{2x_9}+e^{2b}(1+2x_9)}\right] \to \frac{e^{2b}e^{2(x_9-b)}}{2e^{2x_9}} = \frac{1}{2}. \tag{2.28}$$

On the other hand, in the limit $x_9 \to 0, b \to \infty$

$$\left[\frac{e^{2b}(2x_9+e^{2(x_9-b)})}{2e^{2x_9}+e^{2b}(1+2x_9)}\right] \to e^{-2b} \to 0. \tag{2.29}$$

Putting the pieces together,

$$F = 2\pi T_2\frac{1}{2}. \tag{2.30}$$

There is thus a *net force* acting on the membrane, as already mentioned at the end of the previous subsection. In the next section we shall explore why the potential apparently does not cancel for a BPS configuration.

3. Boundary states and extra duality relations

D branes can be represented by boundary states in a closed string theory. Boundary states are BRST invariant states that enforces certain boundary conditions associated with the particular D brane on the string world-sheet ending on it. It is a product of a matter and a ghost part. In the conformal gauge the matter part satisfies the following conditions for a Dp brane [5]

$$\partial_\tau X^l_{\tau=0}|B_X\rangle = 0, \qquad \partial_\sigma X^t_{\tau=0}|B_X\rangle = 0,$$
$$\psi^l - i\eta\tilde{\psi}^l|_{\tau=0}|B_\psi, eta\rangle = 0, \qquad \psi^t - i\eta\tilde{\psi}^t|_{\tau=0}|B_\psi, eta\rangle = 0, \tag{3.1}$$

where l denotes longitudinal directions and t transverse ones. The parameter $\eta = \pm 1$. For an (anti-) brane a unique combination of these states is allowed under GSO projection. A full discussion of the GSO projected states and the ghost states can be found in [5]. The interaction energy between two D branes is thus

$$V = \langle Dp|\Delta|Dq\rangle, \tag{3.2}$$

where Δ is the closed string propagator (4.50) to be discussed in section 4. The NSNS sector gives a finite result [5]

$$V_{\mathrm{NSNS}} = \frac{A}{2\pi}(8\pi^2\alpha')^{-s/2}\int_0^\infty dt\left(\frac{\pi}{t}\right)^{d/2} e^{-L^2/(2\alpha' t)}\left[\left(\frac{f_3}{f_1}\right)^{8-\nu}\left(\frac{f_4}{f_2}\right)^{\nu} - \left(\frac{f_4}{f_1}\right)^{8-\nu}\left(\frac{f_3}{f_2}\right)^{\nu}\right], \tag{3.3}$$

where s is the number of directions shared by the two branes and d is the number of totally Dirichlet directions. The parameter L is the transverse separation of the two branes and $\nu = 8$ is the number of relatively transverse directions. The f_i's are the standard combination of Jacobi θ functions

$$f_1 = q^{\frac{1}{12}}\prod_{n=1}^{\infty}(1-q^{2n}), \qquad f_2 = \sqrt{2}q^{\frac{1}{12}}\prod_{V}^{\infty}(1+q^{2n}),$$
$$f_3 = q^{-\frac{1}{24}}\prod_{n=1}^{\infty}(1+q^{2n-1}), \qquad f_4 = q^{-\frac{1}{24}}\prod_{n=1}^{\infty}(1-q^{2n-1}), \tag{3.4}$$

where $q = e^{-t}$. The RR sector vanishes, however, due to traces of gamma matrices. At the same time the super-ghost sector also diverges. The product of the sectors are regularised, by introducing a regulator [5]

$$x^{F_0+G_0}, \tag{3.5}$$

where F_0 and G_0 are the fermion number operator and super-ghost number operator respectively. The regulator is inserted into all relevant inner products after which the limit $x \to 1$ is taken. It then turns out that the order of the zeroes of the trace of the gamma matrices $\mathrm{tr}(x^{F_0}\Gamma_{11}\Gamma_{i_1}\ldots\Gamma_{i_\nu})$ for $\nu = 8$ cancels the divergence from the divergence arising from the super-ghosts. The resultant RR contribution to the interaction energy is, for $\nu = 8$,

$$V_{\mathrm{RR}} = \frac{A}{2\pi}(8\pi^2\alpha')^{-s/2}\int_0^\infty dt\left(\frac{\pi}{t}\right)^{d/2} e^{-L^2/(2\alpha' t)}(\pm 1). \tag{3.6}$$

Adding the contributions, and making use of the abstruse identity $f_4^8 - f_3^8 + f_2^8 = 0$,

$$V = V_{\mathrm{NSNS}} + V_{\mathrm{RR}} = \frac{V_1}{2\pi\alpha'}L \qquad \text{or} \qquad 0 \tag{3.7}$$

depending on the sign chosen in (3.6), which changes under a parity transformation. The magnitude of the NSNS and RR exchanges each equals one half string tension multiplied by brane separation. This is the manifestation of string creation in perturbative string theory. On the other hand, the non-vanishing RR contribution is surprising because naively a Dp and a D$(8-p)$ brane couples to different RR potentials. To resolve this apparent contradiction, an "off-shell" representation of the RR potentials was proposed in [5] to allow for off-shell momentum exchange between the branes along the totally transverse directions i.e. only k_9 is non-vanishing for the Dp-D$(8-p)$ configuration. Combining with the above regularisation procedure it was found that there is a non-zero overlap between the time-like components of a $p+1$-form and a $9-p$-form. One of the linear combinations of the two components are orthogonal to all other states and the two states could thus be identified. For example

$$|C_0\rangle = -|C_{012345678}\rangle, \tag{3.8}$$

which is very much analogous to the more familiar relations

$$C_p^{\text{transverse}} = \star_8 C_{8-p}^{\text{transverse}}, \tag{3.9}$$

that can also be obtained in this formalism. It was suggested further that since these extra duality relations involve only time-like states which decouple from all string amplitudes, they should not alter the known facts about string theory. We should note that it is not clear how the duality relations should manifest themselves in M-theory, which suffers also from a missing interaction between the M2 brane and KK monopole, as we have analysed in detail previously. Moreover, on closer inspection it is clear that the proposed relation is neither Lorentz invariant nor gauge invariant. Worse still, a D4 brane aligned along x^{01234} should be charged under both C_{01234} and C_{05678} under this proposal. This calls into question the validity of the supergravity solutions obtained for all Dp branes in which they are charged only under one component of the RR potential.

Also, given the off-shell momentum $k_{\mu\neq 0} = 0$, there is no reason to distinguish the "transverse components" from the "longitudinal components". In fact, applying the same procedure to all components C_i, $\langle C_i|C_{012345678}\rangle \neq 0$. It seems that we might need to be more cautious about the interpretation of the duality relations. Recall that the relation (3.9) arises naturally in the light-cone gauge formalisms, where all residual gauge degrees of freedom are fixed. A light-cone gauge has the advantage of retaining only the physical degrees of freedom without the need for ghosts. This suggests the use of light-cone gauge also in attacking this problem. However, as is already known [19], the usual light-cone gauge is incapable of describing the Dp-D$(8-p)$ configurations. This is because by default both light-cone directions become Dirichlet directions whereas the system concerned requires 9 Neumann directions.

4. Closed string generalisation of Arvis gauge

4.1 The bosonic case

The idea of the non-standard light cone gauge makes use of the fact that the left-moving and right-moving modes on the string world-sheet are decoupled from each other for a string theory in a flat background. Therefore we can choose the gauge in each sector independently.

To begin with, the bosonic sector is considered and the gauge choice can be written as

$$\partial_+ X^+ = \alpha' p_+, \tag{4.1}$$

$$\partial_- X^- = \alpha' p_-, \tag{4.2}$$

where we have defined

$$\sigma_\pm = \tau \pm \sigma, \tag{4.3}$$

$$X^\pm = \frac{X^0 \pm X^9}{\sqrt{2}}. \tag{4.4}$$

The transverse directions are given by X^I where $1 \leq I \leq 8$. This can be compared with the gauge condition in the usual light-cone gauge

$$\partial_\pm X^+ = \alpha' p_+. \tag{4.5}$$

The gauge condition, together with the equations of motion in the conformal gauge i.e. the plane-wave equation, imply that the light-cone directions admit the following expansions

$$\begin{aligned} X^+ &= x_0^+ + \alpha' p^+ \sigma_+ + \alpha' \hat{p}^+ \sigma_- + \sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \tilde{\alpha}_n^+ \exp(in\sigma_-), \\ X^- &= x_0^- + \alpha' \hat{p}^- \sigma_+ + \alpha' p^- \sigma_- + \sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \alpha_n^- \exp(in\sigma_+). \end{aligned} \tag{4.6}$$

The hat above p^+ and p^- denotes the fact that they are operators to be determined in terms of the transverse modes $\alpha_n^I, \tilde{\alpha}_n^I$. The gauge conditions have in effect turned off all left-moving X^+ oscillations and right-moving X^- oscillations. With this gauge in place, the Virasoro conditions can be solved

$$T_{++} = \partial_+ X . \partial_+ X = 0 = T_{--} = \partial_- X \partial_- X, \tag{4.7}$$

which gives

$$\begin{aligned} \alpha_n^- &= \frac{1}{2\sqrt{2\alpha'} p_+} \sum_m \alpha_m^I \alpha_{n-m}^I, \\ \tilde{\alpha}_n^+ &= \frac{1}{2\sqrt{2\alpha'} p_-} \sum_m \tilde{\alpha}_m^I \tilde{\alpha}_{n-m}^I. \end{aligned} \tag{4.8}$$

for $n \neq 0$ and the zero-modes yield

$$\begin{aligned} -2p^+ \hat{p}^- + p^I p^I &= \frac{1}{2\alpha'} \sum_{m\neq 0} \alpha_{-m}^I \alpha_m^I, \\ -2p^- \hat{p}^+ + p^I p^I &= \frac{1}{2\alpha'} \sum_{m\neq 0} \tilde{\alpha}_{-m}^I \tilde{\alpha}_m^I. \end{aligned} \tag{4.9}$$

These equations imply that

$$\begin{aligned} p^+ &= \hat{p}^+, \\ p^- &= \hat{p}^-, \end{aligned} \tag{4.10}$$

which are equivalent to the mass-shell condition and level-matching condition. These conditions are also necessary to make the coefficient of σ zero in the Fourier expansion (4.6). This ensures that the fields $X^\pm$ are periodic in σ. This can again be compared to the usual light-cone gauge where the analogous conditions are

$$p^- = \hat{p}^- = \hat{\hat{p}}^-. \tag{4.11}$$

In this gauge, the generators of Lorentz transformation would be given by

$$\begin{aligned}
J^{+-} &= x^+\frac{(p^- + \hat{p}^-)}{2} - x^-\frac{(p^+ + \hat{p}^+)}{2}, \\
J^{-I} &= x^- p^I - x^I\frac{(p^- + \hat{p}^-)}{2} + \frac{\alpha'}{2}\sum_{n\neq 0}\left[\alpha^-_{-n}\alpha^I_n - \alpha^I_{-n}\alpha^-_n\right], \\
J^{+I} &= x^+ p^I - x^I\frac{(p^+ + \hat{p}^+)}{2} + \frac{\alpha'}{2}\sum_{n\neq 0}\left[\tilde{\alpha}^+_{-n}\tilde{\alpha}^I_n - \tilde{\alpha}^I_{-n}\tilde{\alpha}^+_n\right],
\end{aligned} \tag{4.12}$$

Since the anomalies are cancelled separately among the left-movers and the right-movers, these generators give the correct Lorentz algebra.

4.1.1 Spectrum

The spectrum in this gauge yields the same number of degrees of freedom as in the usual light-cone gauge. Consider the massless closed-string states. They are given by

$$\alpha^\mu_{-1}\tilde{\alpha}^\nu_{-1}|0,k\rangle. \tag{4.13}$$

The symmetric traceless part of which gives the gravitational perturbations $g_{\mu\nu}$ while the anti-symmetric part gives the excitations of the 2-form potential $B_{\mu\nu}$. Our choice of gauge then gives

$$K_{+I} = K_{I-} = K_{+-} = 0, \tag{4.14}$$

where $K_{\mu\nu} = g_{\mu\nu} + B_{\mu\nu} + \eta_{\mu\nu}\Phi$ and $\Phi = \mathrm{K}/10$ is the dilaton. Since the conformal gauge on the string world-sheet implies the Lorentz gauge on the potentials, we can solve for the non-physical components by making use of the following

$$\begin{aligned}
-p_- K_{+I} - p_+ K_{-I} + p_J K_{JI} &= 0, \\
-K_{I+}p_- - K_{I-}p_+ + K_{IJ}p_J &= 0.
\end{aligned} \tag{4.15}$$

Solving these equations gives

$$\begin{aligned}
B_{+I} &= -\frac{p_J K_{IJ}}{2p_-}, \\
B_{-I} &= \frac{p_J K_{JI}}{2p_+}, \\
g_{+I} &= \frac{p_J K_{IJ}}{2p_-}, \\
g_{-I} &= \frac{p_J K_{JI}}{2p_+}, \\
g_{+-} + B_{+-} + \delta_{+-}\Phi &= -\frac{p_I p_J g_{IJ}}{2p_+p_-}.
\end{aligned} \tag{4.16}$$

Substituting these expressions into the action and keeping only the physical d.o.f's it turns out that the non-local terms are cancelled between the Einstein-Hilbert action and the kinetic terms for the 2-form potential. The resultant action is simply

$$S = \frac{p^2}{2}\left(g_{IJ}g^{IJ} + B_{IJ}B^{IJ} + \Phi^2\right). \tag{4.17}$$

However, it seems that in this gauge it is impossible to switch off all B and consider pure gravity since B_{+i}, B_{-i} and B_{+-} are also expressed in terms of the transverse components of the metric perturbations. Now consider the field-strengths $H = dB$ when all B_{IJ} are switched off. Using the gauge conditions

$$
\begin{aligned}
H_{+-I} &= p_J \frac{-2p_+p_-\delta_{IK} + p_I p_K}{2p_+p_-} g_{JK}, \\
H_{+IJ} &= \frac{-p_K}{2p_-}(p_J g_{Ik} - p_I g_{JK}).
\end{aligned} \tag{4.18}
$$

and similarly for H_{-IJ}. These equations suggest that in general circumstances there would inevitably be non-trivial H. In systems containing branes with 8 relatively transverse directions a non-trivial H is always induced, as we have seen in section 1. Perhaps this gauge is particularly suited for describing such systems.

4.2 Superstring generalisation

4.2.1 RNS formalism

So far we have been dealing with bosonic string theory. However this choice of gauge can readily be extended to superstring theory. Suppose we work within the RNS formalism and introduce 2d world-sheet Majorana fermions ψ^μ. From the equations of motion

$$
\psi^\mu = \begin{pmatrix} \psi^\mu_-(\sigma_-) \\ \psi^\mu_+(\sigma_+) \end{pmatrix}. \tag{4.19}
$$

The NSNS sector. Anti-periodic boundary conditions imply half-integer modes in the Fourier expansion of the fields. To be consistent with the gauge choice (4.1) we need to set $\psi^+_+ = \psi^-_- = 0$ in the NS sector, which can be neatly written as

$$
\rho^0\psi^0 + \rho^1\psi^1 = 0, \tag{4.20}
$$

where ρ^0, ρ^1 are 2d gamma matrices in a basis as given in [17]. The super-conformal constraints $J_\pm = \psi.\partial_\pm X = 0$ can then be solved as usual for the left and right movers separately.

The RR sector. In the RR sector, periodic boundary conditions allow integer modes in the Fourier expansion of the fields. The zero modes of the ψ^μ generate the ten dimensional Clifford algebra. The ground state of the left movers and right movers are respectively degenerate, each corresponding to a 32 component ten dimensional Majorana spinor. The GSO projection can be imposed as usual accordingly for type IIA and type IIB string theories. Then we impose further the gauge projection consistent with (4.1), namely

$$
\begin{aligned}
\Gamma^+|\Psi_{\rm L}\rangle &= 0, \\
\Gamma^-|\tilde{\Psi}_{\rm R}\rangle &= 0,
\end{aligned} \tag{4.21}
$$

for the left-moving and right-moving ground states respectively. The super-conformal constraint, which corresponds to the Dirac equation, can then be solved.

4.2.2 GS formalism

In order to discuss spacetime supersymmetry, it is more convenient to work with the GS formalism. To obtain $N = 2$ $d = 10$ supersymmetry, we introduce the spacetime spinor coordinates θ^a, which are the 16-component ten dimensional chiral Majorana spinors, where $a \in 1, 2$. Their chiralities are chosen accordingly in type IIA and IIB superstring theories.

The κ-symmetry transformations of the spinor coordinates are given by

$$\delta\theta^a = 2i\Gamma.\Pi_\alpha \kappa^{a\alpha}, \tag{4.22}$$

where

$$\Pi^\mu_\alpha = \partial_\alpha X^\mu - i\bar{\theta}^a\Gamma^\mu\partial_\alpha\theta^a, \tag{4.23}$$

and $\kappa^{a\alpha}$ are 10 dimensional chiral spinors satisfying

$$\begin{aligned} \kappa^{1\alpha} &= P_-^{\alpha\beta}\kappa^1_\beta, \\ \kappa^{2\alpha} &= P_+^{\alpha\beta}\kappa^2_\beta. \end{aligned} \tag{4.24}$$

where $P^{\alpha\beta}_\pm = 1/\left(2\sqrt{h}\right)(h^{\alpha\beta} \pm \epsilon^{\alpha\beta})$. This implies that $a = 1$ describes right-movers while $a = 2$ describes left-movers.

Imposing the gauge condition (4.1) leads to,

$$\Pi^+_+ = p^+, \qquad \Pi^-_- = p^-. \tag{4.25}$$

Studying the equations of motion as listed in [18] suggests that we can consistently set

$$\Gamma^-\theta^1 = 0, \qquad \Gamma^+\theta^2 = 0. \tag{4.26}$$

This choice would keep the gauge choice (4.1) unchanged under κ-transformation of X^μ. Notice that the same projections were obtained for the ground states in RNS formalism. This should also be compared with the projection adopted in the usual light-cone gauge

$$\Gamma^+\theta^1 = \Gamma^+\theta^2 = 0. \tag{4.27}$$

This will have an impact on the spectrum in the RR sector in this gauge.

The combined supersymmetry and κ-transformation that preserves the gauge choice is given by

$$\begin{aligned} \delta X^i &= i\bar{\eta}\Gamma^+\Gamma^i\theta^1\left(\frac{p^-}{4}\right) + i\bar{\eta}\Gamma^-\Gamma^i\theta^2\left(\frac{p^+}{4}\right), \\ \delta\theta^1 &= 2i\Gamma^i\partial_- X^i\eta, \\ \delta\theta^2 &= 2i\Gamma^i\partial_+ X^i\eta. \end{aligned} \tag{4.28}$$

The resultant gauge equations of motion again reduce to $\partial_+\partial_- X^i = \partial_+\theta^1 = \partial_-\theta^2 = 0$.

With the gauge projection condition we can re-write the θ^a's as eight -dimensional chiral spinors, whose chiralities depends on their chiralities in 10 dimensions. Following conventions in [18], we define

$$S^m = \begin{pmatrix} \tilde{S}^m_- \\ S^m_+ \end{pmatrix} = \begin{pmatrix} \sqrt{2p^-}\theta^{1m} \\ \sqrt{2p^+}\theta^{2m} \end{pmatrix}, \tag{4.29}$$

where $m \in \{1,2,3,4,5,6,7,8\}$ are 8-dimensional chiral spinor indices. The effective action that gives the equations of motion is then

$$\int d^2\sigma \frac{-1}{2\pi}(\partial_\alpha X^i \partial^\alpha X^i) + \frac{i}{\pi}(S^m \rho^\alpha \partial_\alpha S^m). \tag{4.30}$$

The bosonic fields admit the same Fourier expansion as in the RNS formalism. The S^m have become world-sheet spinor fields which satisfy periodic boundary conditions. They admit integer modes in Fourier expansion

$$S^m = \frac{1}{\sqrt{2}}\begin{pmatrix} \sum_n \tilde{S}_n^m \exp(in\sigma_-) \\ \sum_n S_n^m \exp(in\sigma_+) \end{pmatrix}. \tag{4.31}$$

Since S^m satisfy the canonical anti-commutation relations, we have

$$\begin{aligned} \{S_h^m, S_k^n\} &= \delta^{mn}\delta_{h+k}, \\ \{\tilde{S}_h^m, \tilde{S}_k^n\} &= \delta^{mn}\delta_{h+k}. \end{aligned} \tag{4.32}$$

The super-charges that generate spacetime supersymmetry transformation (4.28) can now be readily obtained. For concreteness we consider type IIA superstring, in which case θ^1 and θ^2 have the same chirality. The supercharges are

$$\begin{aligned} Q^m &= \sqrt{2p^+} S_0^m, \qquad & Q^{\dot m} &= \frac{1}{\sqrt{p^+}}\gamma^i_{\dot m m}\sum_n S^m_{-n}\alpha^i_n, \\ \tilde{Q}^m &= \sqrt{2p^-}\tilde{S}_0^m, \qquad & \tilde{Q}^{\dot m} &= \frac{1}{\sqrt{p-}}\gamma^i_{\dot m m}\sum_n \tilde{S}^m_{-n}\alpha^i_n. \end{aligned} \tag{4.33}$$

The algebra of the supercharges is

$$\begin{aligned} &\{Q^m, Q^n\} = 2p^+\delta^{mn}, \qquad \{Q^{\dot m}, Q^{\dot n}\} = 2\hat{p}^-\delta^{mn}, \qquad \{Q^m, Q^{\dot n}\} = \sqrt{2}\gamma^i_{m\dot n}p^i, \\ &\{\tilde{Q}^m, \tilde{Q}^n\} = 2p^-\delta^{mn}, \qquad \{\tilde{Q}^{\dot m}, \tilde{Q}^{\dot n}\} = 2\hat{p}^+\delta^{mn}, \qquad \{\tilde{Q}^m, \tilde{Q}^{\dot n}\} = \sqrt{2}\gamma^i_{m\dot n}p^i, \end{aligned} \tag{4.34}$$

where

$$\begin{aligned} -2p^+\hat{p}^- + p^I p^I &= \frac{1}{2}\sum_{m\neq 0}\alpha^I_{-m}\alpha^I_m + nS^m_{-n}S^m_n, \\ -2p^-\hat{p}^+ + p^I p^I &= \frac{1}{2}\sum_{m\neq 0}\tilde{\alpha}^I_{-m}\tilde{\alpha}^I_m + n\tilde{S}^m_{-n}\tilde{S}^m_n. \end{aligned} \tag{4.35}$$

These are again the mass-shell condition and level-matching condition as in (4.9).

4.2.3 Spectrum

Since the zero modes of S^m and $\tilde{S}^{\dot m}$ satisfy the algebra

$$\{S_0^m, S_0^n\} = \delta^{mn}, \qquad \{S_0^{\dot m}, S_0^{\dot n}\} = \delta^{\dot m\dot n}, \tag{4.36}$$

the representation space for the left-movers and right-movers is each given by $8_{\rm v} + 8_{\rm c}$ by triality. The direct product of the massless vectors again give 64 massless states $g + B$

and the dilaton Φ in the gauge as discussed in section (4.1.1). The direct product of the massless spinors also gives 64 massless states. However, they decompose into 8 dimensional even forms, as opposed to odd forms obtained in the usual light-cone gauge.

$$A_{I_1 \ldots I_{2n}} = |\dot{m}\rangle \left(\gamma^{I_1} \ldots \gamma^{I_{2n}}\right)_{\dot{m}\dot{n}} |\tilde{\dot{n}}\rangle. \tag{4.37}$$

This is a result of our opposite projections on the left-moving and right-moving ten dimensional spinors. In the usual light-cone gauge the 1-form and 3-form are interpreted as the transverse physical components of the two-form and four-form field strengths in ten dimensions respectively. It is not clear what should be the correct interpretation of these states in (4.37). However, as we shall see in the next section, an 8d p-form couples to the boundary state corresponding to a Euclidean Dp brane (of $p+1$ dimensions with at least 1d embedded along X^9). Therefore a p-form $A_{I_1 \ldots I_p}$ should probably be interpreted as the $C_{9 I_1 \ldots I_p}$ component of the $p+1$-form potential.

These forms would satisfy a duality relation as a result of 8d Poincare duality. Namely,

$$A_{+I_1 \ldots I_{2n}} = \epsilon_8^{I_1 \ldots I_{2n} J_{2n+1} \ldots J_8} A_{-J_{2n+1} \ldots J_8}. \tag{4.38}$$

This duality condition has been proposed in [5]. A non-zero overlap between the 2 states concerned is obtained where the infinities from the ghosts are shown to cancel the zero from the trace of gamma matrices by employing a regularisation procedure. In our gauge however this relationship is obtained naturally without any divergence appearing. Yet it is important to note that the duality relationship following from our gauge is not gauge invariant. It is really our choice of gauge that leads to this particular relation, much in the same spirit of the usual light-cone gauge where a p form is related to an $8-p$ form. More interestingly, via further Poincare dualities in 10-dimensions, the C_+ component is related to the F_{10} form, which is dual to the cosmological constant. It seems that this gauge could be related to massive supergravity.

On the other hand, our interpretation of these states as components of the potentials would lead to an apparent redistribution of degrees of freedom, where the transverse components of the 1-form potential are "transferred" to the higher forms. It is not clear whether this is related to the Stuckelberg symmetry in massive IIA supergravity.

4.3 Boundary states

We are now in a position to construct boundary states that represent D-Branes. The special gauge choice requires automatically that X^0 satisfies Dirichlet boundary condition while X^9 satisfies Neumann boundary condition. As a result the D-brane is again a Euclidean instanton as in [19] with at least 1d embedded along X^9.

The conditions satisfied by a boundary state representing a Dp-brane transverse to X^t and along X^l are

$$X^t|B,\eta\rangle = 0, \qquad \partial_\tau X^l|B,\eta\rangle = 0, \tag{4.39}$$

$$S^m + i\eta M_{mn}\tilde{S}^n|B,\eta\rangle = 0, \qquad \tilde{S}^{\dot{m}} + i\eta M_{\dot{m}\dot{n}}\tilde{S}^{\dot{n}}|B,\eta\rangle = 0, \tag{4.40}$$

where $\eta = \pm 1$ for brane and anti-brane respectively. The zero-modes of (4.39) give the combination of supercharges that are preserved by the Dp-brane while the other orthogonal combinations are the broken supersymmetries. The matrices $M_{mn}, M_{\dot{m}\dot{n}}$ can be solved for as in [19]. They are given by

$$M_{mn} = (\gamma^1\gamma^2 \dots \gamma^p)_{mn}, \qquad M_{\dot{m}\dot{n}} = (\gamma^1\gamma^2 \dots \gamma^p)_{\dot{m}\dot{n}}. \tag{4.41}$$

The conditions (4.39) can be decomposed into Fourier modes to give

$$\begin{aligned} &\alpha_n + T.\tilde{\alpha}_{-n}|B,\eta\rangle = 0, \\ &S_q^m + i\eta M_{mn}\tilde{S}^n_{-q}|B,\eta\rangle = 0, \qquad S_q^{\dot{m}} + i\eta M_{mn}\tilde{S}^{\dot{n}}_{-q}|B,\eta\rangle = 0 \end{aligned} \tag{4.42}$$

where

$$T = \begin{pmatrix} -I_{p+1} & 0 \\ 0 & I_p \end{pmatrix}. \tag{4.43}$$

The reflections of the transverse modes in (4.42) automatically lead to

$$\alpha_n^- + \tilde{\alpha}^+_{-n}|B,\eta\rangle = 0. \tag{4.44}$$

This implies

$$\alpha_n^0 + \tilde{\alpha}^0_{-n}|B,\eta\rangle = 0, \qquad \alpha_n^9 - \tilde{\alpha}^9_{-n}|B,\eta\rangle = 0. \tag{4.45}$$

The Dp-brane thus lie along X^9 transverse to X^0 as claimed.

The boundary state satisfying the required conditions is then given by [19]

$$|B\rangle = \exp\left(\sum_{q>0}\left(\frac{1}{q}T_{IJ}\alpha^I_{-q}\tilde{\alpha}^J_{-q}\right) - iM_{mn}S^m_{-q}S^n_{-q}\right)|B_0\rangle, \tag{4.46}$$

where

$$|B_{0(I_1\dots I_p,9)}\rangle = [C(T_{IJ}|I\rangle|J\rangle + iM_{\dot{m}\dot{n}}|\dot{m}\rangle|\dot{n}\rangle)]_{p=0}. \tag{4.47}$$

This implies that the boundary state couples to the RR-forms as

$$\langle C_{+I_1\dots I_p}|B_{0(I_1,\dots I_p,9)}\rangle = \mathrm{tr}(I_8). \tag{4.48}$$

The duality relation discussed above also ensures that a Dp brane and a D$(8-p)$ brane couples to the same $p+1$ form, which is exactly the result obtained in [5].

4.4 Tree-level interaction between boundary states

In order to discuss interactions between two branes, a closed string propagator is needed. We have observed in section (4.1) that the coefficient of the linear σ term in the expansion of the X^0, X^9 fields only vanish when both the mass shell condition and level matching condition are satisfied. This raises concern over consistency of introducing off mass shell internal states. On the other hand, for $\partial_\sigma X^0 = \partial_\tau X^9 = 0$ at the boundaries we need only

$$p^+ + \hat{p}^- - \hat{p}^+ - p^- = 0, \tag{4.49}$$

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which is satisfied for $p^9 = 0$ and that the level-matching condition is satisfied. This is indeed the case for the brane configurations under consideration. For the moment we impose the level-matching condition and assume that it is possible to consider off-mass-shell states in a consistent manner.

A closed string propagator is given by [18]

$$\Delta = \int \frac{d^2z}{|z|^2} z^{L_0} \bar{z}^{\tilde{L_0}}. \tag{4.50}$$

The interaction between boundary states is then given by

$$\langle B_{p+1}|\Delta|B_{q+1}\rangle, \tag{4.51}$$

where $|B_{q+1}\rangle$ is as given in (4.46).

If the intermediate state is off-mass-shell, we can write

$$L_0 = p^+(\hat{p}^- - p^-) = -p^+p^- + \frac{p^Ip^I}{2} + N^\perp, \tag{4.52}$$

and similarly for the right-movers. Evaluating the zero- modes part of (4.51), it is given as in usual covariant quantisation. The bosonic sector, for example, would give

$$\langle p=0, I_L|\langle p=0, J_R|T_{IJ}\delta^{d_\perp}(\hat{x}_i)|z|^{\alpha' p^2/2}\delta^{d'_\perp(\hat{x}_i - y_i)}|p=0, H_L\rangle|p=0, K_R\rangle T'_{HK}. \tag{4.53}$$

Expressing the δ functions as integrals the above expression can be simplified to

$$W\int \frac{d^{d_{\perp Dp+Dq}}Q}{(2\pi)^{d_{\perp Dp+Dq}}}|z|^{\alpha' Q^2/2}\exp{(iQ.y)}\mathrm{tr}(TT'), \tag{4.54}$$

where W is some normalisation constant proportional to the volumes of the branes and we have used

$$\langle p, I|q, J\rangle = 2\pi\delta(p-q)\delta_{IJ}. \tag{4.55}$$

The non-zero modes involve only the transverse components and can be evaluated as usual. In the special case where the two boundary states have 8 relatively transverse dimensions, the non-zero modes give

$$\frac{f_2^8(|z|)}{f_2^8(|z|)}, \tag{4.56}$$

The bosonic zero-modes are proportional to $\mathrm{tr}(-I_8) = -8$ and the fermionic zero-modes are proportional to

$$\mathrm{tr}(\gamma^1 \ldots \gamma^8) = \pm 8. \tag{4.57}$$

The sign depends on the projection of the ten dimensional chiral spinor in equation (4.26). Upon a parity change along X^0 we exchange Γ^+ and Γ^- and this would switch the sign of equation (4.57). This is in support of the well-known fact that when the two branes cross each other a string has to be created. This also implies that D8 brane's coupling to the C_9 potential changes sign under parity. This is in agreement with the result [20] that D8 brane behaves like a domain wall in massive IIA SUGRA across which the cosmological constant jumps. It is perhaps beyond perturbative string theory to see how the cosmological constant jumps in steps as we cross more than two D8 branes. This is because each closed string world-sheet can at most connect to two boundaries.

5. The no force condition in the low energy effective field theory

So far we have been analysing the system in the closed string channel in which the branes interact via tree level closed string exchange. The same system, however, can be described by open-strings, which we will discuss in the next subsection. This will lead us to consider the effective field theory in section 5.2.

5.1 Open-string description of the Dp-D$(8-p)$ system

The closed string tree level interaction potential between the branes is equivalent to the open-string one-loop Coleman-Weinberg effective potential by a modular transformation. These open-strings concerned are stretched between the two sets of branes. According to the boundary conditions satisfied by the world-sheet supersymmetry current, they are divided into two sectors, the NS-sector if anti-periodic and the R-sector if periodic. The Coleman-Weinberg potential is given by $\log\det(\partial^2 + M^2) = \mathrm{tr}\log(\partial^2 + M^2)$. The trace is turned into an integral over all momenta, and also runs over all states in the R and NS sector after GSO projection. The integral over momenta can be turned into an integral over the Schwinger parameter t. The potential is most easily evaluated in the light-cone gauge (or the Arvis gauge for the system concerned) with all the unphysical states removed. The calculation is reviewed in appendix A where the potential is given by (A.5). The projector $(1 + (-1)^{F_0}\Gamma_{\mathrm{R,NS}})$ inserted in the trace implements the GSO projection explicitly. The traces without the Gamma matrices in the R and NS sector i.e. $\frac{1}{2}\mathrm{tr}_{\mathrm{NS}} q^{L_0 - a_{\mathrm{NS}}}$ and $\frac{1}{2}\mathrm{tr}_{\mathrm{R}} q^{L_0}$, adds up and contribute to a term

$$\frac{HK}{2 \times 2\pi\alpha'}|L| \tag{5.1}$$

in the effective potential, where H, K are the number of branes on each stack and L the separation. As long as the branes are not parallel to each other, there exist directions parallel to one set of the branes and transverse to the other. In which case, the NS sector fermions along those directions carry zero modes, so that $\mathrm{tr}_{\mathrm{NS}}\Gamma_{\mathrm{NS}} = 0$ along those directions. In all other supersymmetric D-branes configurations, the branes share more than two common longitudinal or transverse directions. Therefore, for strings stretched between the two sets of branes, after removing the light-cone directions there still exist world-sheet fermions having the same boundary conditions on both end-points and carry zero modes. As a result $\mathrm{tr}_{\mathrm{R}}\Gamma_{\mathrm{R}} = 0$. This however is not true for Dp-D$(8-p)$ systems in which there are exactly one common longitudinal direction and one common transverse direction. Modes along those directions can be removed after gauge fixing. Along all other directions the world-sheet fermions do not carry zero modes and $\Gamma_{\mathrm{R}} = 1$ in the light-cone gauge, thus giving a non-vanishing contribution to the Coleman-Weinberg potential. In the conformal gauge however, this term is resulted from the cancellation between the zero arising from the world-sheet fermionic zero modes and the divergence from ghost zero modes [3], which is explained below.[2] In fact it contributes to a potential

$$\frac{HK}{2 \times 2\pi\alpha'}L. \tag{5.2}$$

[2] The situation is similar to that in the closed string RR exchange calculation.

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This extra piece that is absent in all other supersymmetric brane configurations can be identified with the unexpected RR exchange in equation (3.6) under modular transformation in the closed string channel discussed in previous sections.

The physical interpretation of this term is not clear. As we will discuss further in the next subsection, it does not have a microscopic origin in the low energy effective field theory where only the physical spectrum predicted in string theory is taken into account. So far this term has been added in by hand to make the theory consistent and supersymmetric. But a direct correspondence between string theory and the effective field theory must imply that the term could be understood entirely in the effective field theory framework. We therefore would like to identify the precise string states contributing to this term in order to give it a microscopic interpretation in the effective field theory.

The partition sum of all the sectors in (A.6) is a constant independent of the parameter t. This implies that it receives contribution only from the world-sheet ground states. To find out the precise ground states, including possible contributions from non-physical states in the R sector, we should work again in the conformal gauge. We must combine the contributions of the (R,NS) and (R,R) sector i.e. the two parts that constitute the GSO projection, rather than viewing them independently. In other words, we are interested in the world-sheet fermions and $\beta\gamma$ zero modes contribution to the partition sum

$$F_{\mathrm{R}} = F_{(\mathrm{R,NS})} \pm F_{(\mathrm{R,R})}, \tag{5.3}$$

where

$$F_{(\mathrm{R,NS})} = \frac{1}{2}\mathrm{tr}_{\mathrm{R}}\left((-x)^{G_{\beta\gamma}} q^{2N}\right), \qquad F_{(\mathrm{R,R})} = \frac{1}{2}\mathrm{tr}_{\mathrm{R}}\left((-y)^{F}(x)^{G_{\beta\gamma}} q^{2N}\right),$$
$$q = \exp(-\pi t). \tag{5.4}$$

A priori the term $F_{\mathrm{R,R}}$ is ill-defined due to divergent sums coming from the infinite number of ghost contributions. The quantities x, y regulate these divergences [3, 5] and in the end we must set $x, y = 1$. We will soon argue that consistency also requires $x = y$. The notation used here and ghost quantisation in the conformal gauge are explained in appendix B. The $\pm$ sign in (5.3) reflects the ambiguity in defining the GSO projection. Without loss of generality we consider the case where there is exactly one brane on each stack. Using the results in appendix B and keeping track explicitly of the contribution of the two world-sheet fermion ground states $|\downarrow\rangle, |\uparrow\rangle$ in the R sector, the zero mode contribution of the first term in (5.3) is

$$F^{(0)}_{(\mathrm{R,NS})} = \frac{\langle\uparrow|\uparrow\rangle + \langle\downarrow|\downarrow\rangle}{2}\frac{1}{1+x}. \tag{5.5}$$

Since $\langle\uparrow|\uparrow\rangle = \langle\downarrow|\downarrow\rangle = 1$, $F_{(\mathrm{R,NS})} = 1/2$ in the limit mentioned above. The second term in (5.3) is more subtle because it involves the divergence of the ghost zero modes. It is given by

$$F^{(0)}_{(\mathrm{R,R})} = \frac{\langle\downarrow|\downarrow\rangle - y\langle\uparrow|\uparrow\rangle}{2}\frac{1}{1-x}. \tag{5.6}$$

This expression is highly constrained by requiring consistency with Gauss' law. This requires that the value of the potential should jump by one unit of string tension as one brane crosses the other, as in the case in [1]. In other words the potential must jump by one unit when the sign of the GSO projection in (5.3) is flipped. This implies that $F_{(\mathrm{R},\mathrm{R})}$ should take the value 1/2, which, in turn requires us to choose

$$x = y \to 1. \tag{5.7}$$

Since supersymmetry requires vanishing of the potential between the branes, the open string calculation again implies the creation of a string as the branes cross, to cancel the effect of the jump in potential. Closer inspection of F_{R} reveals the precise states that are circulating in the loop. Consider choosing the upper sign in equation (5.3) as a concrete example. We will substitute $y = 1$ directly since it will not affect our following discussion. We will, however, be keeping the sum as a polynomial in x^k where the index k is equal to the ghost number. The zero mode contribution to the partition sum $F_{\mathrm{R}}^{(0)}$ is then given by

$$\begin{aligned} F_{\mathrm{R}}^{(0)} &= \frac{1}{2}\sum_{n=0}^{\infty}(-x)^n(\langle\uparrow\mid\uparrow\rangle + \langle\downarrow\mid\downarrow\rangle) \pm x^n(-\langle\uparrow\mid\uparrow\rangle + \langle\downarrow\mid\downarrow\rangle) \\ &= -\frac{1}{2}\sum_{n=0}^{\infty} x^n\Big(\langle\downarrow\mid\downarrow\rangle(1\pm(-1)^n) - \langle\uparrow\mid\uparrow\rangle(1\mp(-1)^n)\Big) \\ &= -\sum_{n=0}^{\infty}\Big(x^{2n}\langle\downarrow\mid\downarrow\rangle - \langle\uparrow\mid\uparrow\rangle x^{2n+1}\Big). \end{aligned} \tag{5.8}$$

Out of these states only the $n = 0$ term $\langle\downarrow\mid\downarrow\rangle$ came from the physical state $\mid\downarrow\rangle$, which is annihilated by β_0. The rest have non-zero ghost excitations of $2n$ or $2n+1, n > 0$. The lowest physical state is thus a single fermion, as expected. The rest are two infinite towers of ghost states of *opposite* chiralities.[3] Changing the choice of GSO projection flips the chirality of the lowest physical fermion and swaps the two ghost towers.

5.2 Local supersymmetry in quantum mechanics

As we have seen, the closed string tree level calculation of the interaction between the branes can be recast entirely in the open-string language. The potential energy should correspond to the Coleman-Weinberg potential in effective field theory. However, while the open and closed string calculations agree, they clearly do not agree with a naive calculation in field theory, with field content consisting of the physical ground states of the open strings. Without loss of generality, consider the low energy effective field theory of a D0 and K D8 branes. This system is consistent only in the context of type I' theory in the presence of two orientifold planes. But we could always take the limit in which 8 D8 branes and their images lie on each of the orientifold planes and the planes are widely separated. Then we can consider taking K D8 branes from one of the orientifolds and stay sufficiently far from either of the orientifold planes. The effective action of the system is then that of $(0, 8)$

[3]Chirality is understood from the perspective of the T-dual theory where the branes intersect over 1+1 dimensions.

quantum mechanics given as in [7, 22]. In the presence of a single D0 and K D8 branes, there are K "chiral fermions" inert under the unbroken supersymmetries,[4] whose action is

$$\mathcal{L}_\sigma = \sum_r^K \left[-i\sigma_r^\dagger \dot{\sigma}_r - \sigma_r^\dagger (Y + A_0)\dot{\sigma}_r \right], \tag{5.9}$$

where A_0 is the difference between the gauge fields on the D0 and the D8 branes. The fermions are in the fundamental representation of $\mathrm{U}(K)$. The field $Y = Y^{\mathrm{D0}} - Y^{\mathrm{D8}}$ is the separation between the branes. Integrating these fermions out gives [7]

$$\frac{K}{2}|A_0 + Y|, \tag{5.10}$$

which is a result of the fact that for fermions, only the retarded Green's function could be defined in a $0+1$ dimensional field theory. In other words, this is the energy of K fermionic harmonic oscillators each occupying the filled state, where $(Y + A_0)$ is treated as the natural frequency of the oscillators. Comparing with the open string calculation, (5.10) corresponds to the contribution from $\frac{1}{2}\mathrm{tr}_{\mathrm{NS}} q^{L_0 - a_{\mathrm{NS}}}$ and $\frac{1}{2}\mathrm{tr}_{\mathrm{R}} q^{L_0}$ as in equation (5.1). There is an extra piece in the string calculation, coming from

$$\frac{1}{2}\mathrm{tr}_{\mathrm{R}}(-1)^{F_0}\Gamma_{11} q^{L_0}, \tag{5.11}$$

which gives a term $\frac{K}{2}(A_0 + Y)$, as discussed before equation (5.2). From the point of view of effective field theory, the origin of this term is not immediately clear just by inspecting the physical spectrum. It is known just that $K/2(A_0 + Y)$ is needed to render the theory consistent and supersymmetric. It is called the *bare* term and is usually added in by hand.[5] By inspecting the open string calculation in detail in the previous subsection, we concluded that this term arose from the zero modes of two sets of $\beta\gamma$ superghosts. These ghosts must also emerge naturally in the effective field theory, since each state in string theory corresponds to some space-time particle, and the assertion extends to the ghost sector. For example, the level one massless bc ghosts states $b_{-1}|0\rangle$ and $c_{-1}|0\rangle$ in open bosonic string theory indeed correspond to the Fadeev-Popov ghosts needed for removing the non-unitary components of the massless gauge vector. The two towers of ghost states that we have found must also correspond to the infinitely degenerate ground states of two sets of $\beta\gamma$ ghost fields with masses of opposite signs in the effective field theory. The $\beta\gamma$ ghosts are needed in local super-reparametrisation symmetry on the world-line of the R-sector fermionic particle. Note that this is a target space symmetry i.e. a symmetry of the $0+1$ dimensional effective field theory and should not be confused with the string world-sheet supersymmetry. However, given that the fermionic particle is actually the lowest mode of a macroscopic stretched string, its effective action can be thought of as following from dimensional reduction of a two dimensional string world-sheet. This explains the presence of local supersymmetry on its worldline.

[4] Again these are related to chiral fermions in $1+1$ dimensions by a T-duality.

[5] We thank D. Tong for pointing out that the appearance of these terms can be understood as a result of "integrating the chiral fermions in", starting with a manifestly supersymmetric theory with no potential terms.

To demonstrate the presence of these ghosts, we would like to construct the action of the Ramond fermions so that it is explicitly invariant under local super-reparametrisation. Upon gauge fixing the matter part of the action should reduce to equation (5.9). There will be new terms in the action for the $\beta\gamma$ ghosts of the form $\beta G\gamma$, where G is a differential operator that generates the transformation of the gravitini. Without loss of generality we consider $K = 1$. The coupling to the gauge fields look like a mass term and in general it would take the form $ef\bar{\sigma}\sigma$, where e is the world-line metric and f will eventually be identified with $Y + A_0$. Since in general $\delta_\epsilon e \sim i\epsilon\chi$ under supersymmetry transformation, where χ is the gravitino, the variation of the mass term can be cancelled by a term proportional to $\chi^2\bar{\sigma}\sigma$, if

$$\delta_\epsilon \chi \sim \dot{\epsilon} - if\epsilon. \tag{5.12}$$

We shall now derive this transformation rule for the gravitini and justify the claim using superspace method. It is important to note that the second term in this transformation is crucial since it gives rise to the coupling between f and the $\beta\gamma$ ghosts upon gauge fixing the gravitini, as is suggested by the string calculation. Such transformations for the gravitini are familiar in gauged supergravity theories. The gauge fixed local supersymmetry is generally spontaneously broken in these theories since further global supersymmetry transformations generally take the gravitini away from the gauge condition. Also the presence of a term proportional to χ^2 in the action can only arise when there is more than one supersymmetry such that χ is complex. In fact we need exactly two local supersymmetries with a gauged U(1) R-symmetry.

To set the scene we would first review the superspace formalism in super quantum mechanics with two supercharges [23]. The superspace has three coordinates $(\tau, \theta, \bar{\theta})$. The super-vierbein fields E_M^A satisfy

$$E_B^M E_M^A = \delta_B^A. \tag{5.13}$$

The indices $M = (\tau, \theta, \bar{\theta})$ are curved space indices and $A = (t, a, \bar{a})$ are tangent space indices. The super-vierbeins transform as

$$\delta_\zeta E_M^A = \zeta^N \partial_N E_M^A + \partial_M \zeta^N E_N^A, \tag{5.14}$$

and a general scalar super-field Φ transforms as

$$\delta_\zeta \Phi = \zeta^N \partial_N \Phi. \tag{5.15}$$

Also, under tangent space rotations,

$$\delta E_M^a = -E_M^t \phi^a + E_M^b \epsilon_b^a T, \tag{5.16}$$

where a here corresponds to any of the two tangent space fermionic indices, and ϕ^a and T are fermionic and bosonic parameters respectively, of the U(1) transformations. The other components of the super-vierbein do not transform, nor does a scalar super-field. To limit the invariance to the usual supergravity form of super quantum mechanics, the super-vierbein should be partly gauge fixed so that they satisfy

$$E_M^t = \Lambda \bar{E}_M^t, \qquad E_M^a = \Lambda^{1/2} \bar{E}_M^a, \tag{5.17}$$

where $\bar{E}_M^A$ is the flat vierbein given by

$$\bar{E}_M^A = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{i}{2}\bar{\theta} & 1 & 0 \\ -\frac{i}{2}\theta & 0 & 1 \end{pmatrix}, \tag{5.18}$$

and Λ is

$$\Lambda = e + \frac{i}{2}\left(\theta\bar{\chi} + \bar{\theta}\chi\right) + \bar{\theta}\theta\frac{f}{2}. \tag{5.19}$$

The residual symmetries that leave the gauge choice invariant are

$$\begin{aligned}
\zeta^\tau &= a + \frac{i}{2}\left(\theta\bar{\beta} + \bar{\theta}\beta\right), \\
\zeta^\theta &= \beta + \frac{1}{2}\theta\dot{a} - i\theta t + i\theta\bar{\theta}\frac{\dot{\beta}}{2}, \\
\zeta^{\bar{\theta}} &= \bar{\beta} + \frac{1}{2}\bar{\theta}\dot{a} + i\theta t - i\theta\bar{\theta}\frac{\dot{\bar{\beta}}}{2}, \\
T &= t + \frac{1}{2}\left(\theta\dot{\bar{\beta}} - \bar{\theta}\dot{\beta}\right) + \frac{1}{4}\theta\bar{\theta}\ddot{a}, \\
\phi^\theta &= \Lambda^{-1/2}\zeta^\theta, \qquad \phi^{\bar{\theta}} = \Lambda^{-1/2}\zeta^{\bar{\theta}},
\end{aligned} \tag{5.20}$$

where $a, \beta, \bar{\beta}, t$ are arbitrary functions of τ and correspond to general coordinate transformations in τ, local supersymmetry and local U(1) respectively. The field Λ transforms as

$$\delta_\zeta\Lambda = \zeta^M\partial_M\Lambda + \partial_\tau\tilde{\zeta}^\tau\Lambda, \tag{5.21}$$

where

$$\tilde{\zeta}^A = \zeta^M\bar{E}_M^A. \tag{5.22}$$

As we see, Λ transforms with an inhomogeneous term in addition to a homogeneous transformation as a usual scalar super-field.

Then we would introduce the fermionic chiral super-field Ψ satisfying

$$D_{\bar{\theta}}\Psi = 0, \tag{5.23}$$

where $D_A = \bar{E}_A^M\partial_M$ and in particular

$$D_{\bar{\theta}} = \partial_{\bar{\theta}} + \frac{i}{2}\theta\partial_\tau. \tag{5.24}$$

In component form,

$$\Psi = \psi + i\theta F + i\theta\bar{\theta}\eta. \tag{5.25}$$

Solving the condition (5.23) gives $\eta = \dot{\psi}/2$. Eventually we would gauge fix Λ such that $(e, \chi, f) = (1, 0, Y + A_0)$. The correct action for Ψ that reduces to the form (5.9) is

$$S = \int d\tau d^2\theta\Lambda\Psi^\dagger\Psi = \int d\tau\left[-ie\psi^\dagger\dot{\psi} + eF^\dagger F + \frac{1}{2}(\chi^\dagger\psi F - \chi\psi^\dagger F) + \frac{f\psi^\dagger\psi}{2}\right]. \tag{5.26}$$

Clearly F is an auxiliary field and after removing it via the equations of motion the action simplifies to

$$S = \int d\tau \left(-ie\psi^\dagger \dot{\psi} + \frac{f\psi^\dagger\psi}{2} + \frac{\bar{\chi}\chi\psi^\dagger\psi}{4e} \right). \tag{5.27}$$

The action contains the χ^2 term as anticipated at the beginning of the section. However, equation (5.26) also makes it clear that Ψ does not transform simply as in (5.15) since Λ transforms non-trivially. To find the correct transformation rule for Ψ, consider $(\delta_\zeta \Lambda)\Psi^\dagger\Psi$. This can be written as

$$\begin{aligned}(\delta_\zeta \Lambda)\Psi^\dagger\Psi &= (\tilde{\zeta}^A D_A \Lambda + \partial_\tau \tilde{\zeta}^\tau \Lambda)\Psi^\dagger\Psi \\ &= \partial_\tau(\tilde{\zeta}^\tau \Lambda \Psi^\dagger\Psi) - D_a(\tilde{\zeta}^a \Lambda\Psi^\dagger\Psi) + \Lambda\left[\dot{\tilde{\zeta}}^\tau - \dot{\tilde{\zeta}}^\tau + (D_a\tilde{\zeta}^a) - \tilde{\zeta}^A D_A\right](\Psi^\dagger\Psi).\end{aligned} \tag{5.28}$$

From these expressions, we conclude that the action (5.26) is invariant up to total derivative terms if Ψ transforms as

$$\delta_\zeta \Psi = -D_{\bar{\theta}}\tilde{\zeta}^{\bar{\theta}}\Psi + \tilde{\zeta}^A D_A \Psi. \tag{5.29}$$

Since $D_{\bar{\theta}}^2 = D_\theta^2 = 0$ this set of transformations explicitly preserve the constraint (5.23). We could further rescale the fields and define

$$\begin{aligned}\Xi &= \frac{\chi}{\sqrt{e}}, \qquad e(Y + A_0) = \frac{f}{e}, \\ \sigma &= \sqrt{e}\psi,\end{aligned} \tag{5.30}$$

so that under reparametrisation,

$$\begin{aligned}\delta\left(e, \Xi, e(Y + A_0)\right) &= a\partial_\tau\left(e, \Xi, e(Y + A_0)\right) + \dot{a}\left(e, \Xi, e(Y + A_0)\right), \\ \delta\sigma &= a\dot{\sigma}.\end{aligned} \tag{5.31}$$

In these new variables, the gravitini transforms as

$$\delta\Xi = 2\dot{\alpha} - i\alpha e^2(Y + A_0) - i\alpha\frac{\bar{\Xi}\Xi}{4e} - it\Xi, \qquad \delta\bar{\Xi} = 2\dot{\bar{\alpha}} + i\bar{\alpha}e^2(Y + A_0) + i\bar{\alpha}\frac{\bar{\Xi}\Xi}{4e} + it\bar{\Xi}, \tag{5.32}$$

where $\alpha = \beta/\sqrt{e}$. Gauge fixing Ξ and $\bar{\Xi}$ to zero then gives rise to ghost actions

$$\beta(2\partial_\tau \pm ie^2(Y + A_0))\gamma. \tag{5.33}$$

Note that automatically we need two sets of ghosts of opposite masses as is required. Under T-duality along the Y direction where we exchange $Y + A_0$ for the momentum k_Y, these ghosts have opposite chiralities in the effective theory living in the 1+1 dimensional intersection domain of the branes. They should be compared with gauge fixing of the world-sheet local supersymmetries in usual superstring theories. These ghosts are the origin of the *bare* CS term and mass term as is found in the open-string calculation. We have considered the special case where $K = 1$ where there is only one complex Ramond ground state fermion. For general K we have K complex fermions. Each of these fermions possesses local supersymmetries and thus give rise to $2K$ sets of superghosts. It is now clear that the extra piece in the effective potential can be understood entirely in the framework of effective field theory, and whose coefficient is uniquely fixed, by the coupling of the gauge theory to a 0+1 supergravity theory. Interestingly, our construction also implies that the gauge field to which the fermion couples gauges the R-symmetry of the spontaneously broken local supersymmetries.

5.3 A remark on half-strings

We have learnt from the probe calculation that if the mysterious RR exchange is ignored a Dp probe apparently experiences a nontrivial potential in a D$(8-p)$ background. The RR exchange generates a balancing force that was interpreted as the effect of a *half-string* in [3, 8]. There are other arguments that have supported this notion of a half-string [7, 8]. Taking again the example of the Dp probe brane and examining the Chern-Simons terms, there is a term $\mu_p \int_{\mathcal{M}_{p+1}} C_{p-1} \wedge F$ (for $p \leq 8-p$), which, upon integration by parts, becomes

$$\mu_p \int_{\mathcal{M}_{p+1}} H_p \wedge A, \tag{5.34}$$

where $\mathcal{M}_{p+1}$ is the manifold spanned by the probe world-volume . Since the background D$(8-p)$ brane is magnetically charged with respect to H_p, it can be integrated over the hemisphere h_p, transverse to the background brane, upon which the world-volume of the non-compact probe wraps , yielding $\frac{1}{2}\mu_{8-p}$. The factor of $\frac{1}{2}$ arises because the integral is over a hemisphere. Equation (5.34) thus becomes

$$\frac{1}{2}\mu_p \mu_{8-p} \int A = \frac{1}{4\pi\alpha'} \int A. \tag{5.35}$$

This is half of the charge carried by the end-point of the string on the brane induced by the background flux. Furthermore, as discussed in the previous subsection the K complex fermionic fields, in $N=8$ quantum mechanics, gave rise to a CS term $\frac{K}{2}A_0$ upon being integrated out. The *bare* term is needed so that the overall coefficient of the CS term is integer valued for consistency. This bare term is interpreted as arising from (5.34) where half-strings supply the extra fractional charges.

In perturbative string theory however, there is no notion of fractional string states. These closed string tree-exchange, can be understood as loops in the open string channel, in which usual open string states circulate. It should be clear from our analysis in the last subsection that every term in the low energy effective field theory arises from usual open-string states.

6. Conclusion

In this work we have reviewed the problem of string creation and the remaining questions that have not yet been satisfactorily answered. Namely, that while a perturbative string calculation yields a vanishing potential (or a potential due to a stretched string) between a pair of Dp and D$(8-p)$ branes a probe calculation gives a non-zero potential. The problem persists to M-theory. To yield a vanishing potential, it was suggested that there exist half

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strings connecting the branes. On the other hand, a careful analysis in [5] suggests a surprising non-zero contribution from the RR sector hitherto not considered. However, extra duality relations, which are neither Lorentz invariant nor gauge invariant, were obtained to explain the unexpected coupling. These calculations involve a regularisation procedure to tame the divergences from the ghost sector. To avoid the use of ghosts which potentially causes confusion, we propose gauge fixing in a light-cone like gauge. It is a direct closed string version of Arvis' gauge [4] and is capable of describing these Dp-D$(8-p)$ systems. The same conclusion as in [5], of a non-zero RR exchange is reached without encountering divergences. However, the gauge makes it clear that the extra duality relations are in fact artifacts of our special gauge choice. Moreover, in order that the closed string propagator makes sense, the momentum exchanged along p_9 has to vanish in the Euclidean picture, which corresponds to p_0 in the Minkowskian picture. This suggests that the time direction is special and further Lorentz symmetry is broken before such exchange is possible. This means that the supergravity solutions of parallel Dp branes should not be affected by this apparent extra coupling to a different RR potential. Finally we discuss how the mismatch in the effective potential between the effective field theory calculation and the corresponding open-string theory calculation can be reconciled. The extra piece in string theory is attributed to contributions from $\beta\gamma$ ghosts zero modes. The piece is known as the *bare* term in the field theory, added in by hand for consistency and supersymmetry. We show that as in string theory, this term originates from ghosts for gauge fixing spontaneously broken local world-line supersymmetry of the Ramond ground state fermion. In any case, the half-tensions arose from quantum effects. The term *half-strings* is perhaps a misnomer since a classical stringy object with half of the tension of a fundamental string need never be introduced.

A. Arvis gauge and open string loop

A closed string tree exchange can be mapped to a 1-loop open string diagram by a suitable conformal transformation. More intuitively, the dynamics of D branes can be described by open strings and the interaction energy between two D branes can be obtained from the 1-loop vacuum diagram of an open string connecting the two D branes, using the Coleman-Weinberg formula. To avoid the use of ghosts, we will introduce the Arvis gauge [4] which is a form of light-cone gauge where the "light-cone" directions x_0 and x_9 are allowed to satisfy different boundary conditions.

In the original context, the gauge was used for describing the interaction of two D-particles separated along x_9. It is thus convenient to introduce a gauge such that x_0 satisfies Neumann boundary condition on both ends of the open string whereas all other coordinates should satisfy Dirichlet boundary condition. This implies that in the conformal gauge the world-sheet fields admit the following Fourier expansion [4]

$$\begin{aligned} x^i(\sigma,\tau) &= R^i\sigma + \sqrt{2}\sum_{n\neq 0}\frac{a_n^i}{n}\sin n\sigma e^{-in\tau}, \\ x^0(\sigma,\tau) &= q_0 + p^0\tau + i\sqrt{2}\sum_{n\neq 0}\frac{a_n^0}{n}\cos n\sigma e^{-in\tau}. \end{aligned} \tag{A.1}$$

Further gauge conditions

$$\begin{aligned}\dot{x}^9 + x'^0 &= 0,\\ \dot{x}^0 + x'^9 &= p^0 + R,\\ q^0 &= 0,\end{aligned} \tag{A.2}$$

are subsequently imposed, which is equivalent to setting

$$a_n^+ \equiv a_n^0 + a_n^9 = 0. \tag{A.3}$$

The Virasoro conditions can then be solved explicitly such that $a_n^- \equiv a_n^0 - a_n^9$ can be expressed in terms of the rest of the transverse oscillators. The Lorentz generators can be built accordingly. Clearly x^0 is different from the components x^i due to boundary conditions. For bosonic strings it can be shown that the system enjoys SO(25) rotational invariance for $d = 26$. This can be readily generalised to open superstring theory. The gauge condition (A.2) can be written as

$$\partial_+ x^+ = \partial_- x^- = p^0 + R \tag{A.4}$$

which will make generalisation to closed string pursued in the next section more transparent.

The interaction energy between a Dp and a D$(8-p)$ string is then given by

$$\begin{aligned}\int_0^\infty \frac{dt}{2t}(8\pi^2\alpha' t)^{-1/2} e^{-L^2 t/(2\pi\alpha')} \mathrm{tr}_R \left(\frac{1+(-1)^{F_0}\Gamma_{\mathrm{R}}}{2} e^{-2\pi t(L_0 - a_{\mathrm{R}})}\right)\\ + \mathrm{tr}_{NS}\left(\frac{1+(-1)^{F_0}\Gamma_{\mathrm{NS}}}{2} e^{-2\pi t(L_0 - a_{\mathrm{NS}})}\right),\end{aligned} \tag{A.5}$$

where L_0 is the zero mode of the Virasoro operator, $a_{\mathrm{R,\ NS}}$ are the vacuum contributions to the energies in the respective sector and $\frac{1+(-1)^{F_0}\Gamma_{\mathrm{R,NS}}}{2}$ are the GSO projection in the respective sectors. The Gamma matrices $\Gamma_{\mathrm{R,NS}}$ are built from the zero modes of the world-sheet fermions. The operator F_0 is again the world-sheet fermion number operator. It will be convenient to define $q = \exp(-\pi t)$. When there are exactly 8 relatively transverse directions, all transverse world-sheet fermions in the Ramond sector obtain half-integer moding but integer moding in the NS sector. Therefore $\mathrm{tr}((-1)^{F_0}\Gamma_{\mathrm{NS}})$ vanishes but $\mathrm{tr}((-1)^{F_0}\Gamma_{\mathrm{R}})$ gives a non-zero contribution.

Adding the contributions together the potential between the branes is given by

$$-\int_0^\infty \frac{dt}{2t}(8\pi^2\alpha' t)^{-1/2} e^{-L^2 t/(2\pi\alpha')}\left[\frac{-f_2^8 + f_3^8 \pm f_4^8}{f_4^8}\right] = -\frac{1}{2}T_1 L[1 \pm 1]. \tag{A.6}$$

The third term originates from $\mathrm{tr}((-1)^{F_0}\Gamma_{\mathrm{R}})$. The choice of signs corresponds to the possible choices of GSO projections, usually interpreted as the different projections required for branes or anti-branes. However, opposite GSO projections corresponds to a flipped relative orientation of the branes and is equivalent to the two branes crossing each other. Therefore we are led to the same conclusion as in the closed string calculation that a string has to be created whenever the branes cross each other.

B. Open string partition sum in conformal gauge and regularisation

A crucial part in the calculation of the interaction potential between the branes is to evaluate the partition sum

$$\mathrm{tr}\left(\frac{(1+(-1)^F)}{2}q^{2N}\right). \tag{B.1}$$

The sum is straight forward in Arvis gauge as is presented in the previous section. In conformal gauge, the residual symmetry requires the introduction of ghosts. The partition sum should therefore also include their contribution. The residual Virasoro symmetry calls for a pair of anti-commuting bc ghosts and the world-sheet supersymmetry calls for a pair of commuting $\beta\gamma$ ghosts. These ghosts can be expanded in Fourier modes and be quantised. Their modes satisfy the following (anti-)commutation relations [18]

$$\begin{aligned}
b_{++} &= \sum_n b_n \exp -in\sigma_+, & c^+ &= \sum_n c_n \exp -in\sigma_+,\\
\beta_{3/2} &= \frac{1}{\sqrt{2}}\sum_n \beta_{\nu+n} \exp -i(n+\nu)\sigma_+, & \beta_{-1/2} &= \frac{1}{\sqrt{2}}\sum_n \gamma_{\nu+n} \exp -i(n+\nu)\sigma_+\\
\{c_s, b_t\} &= \delta_{s+t}, & \{c_s, c_t\} &= \{b_s, b_t\} = 0,\\
[\gamma_s, \beta_t] &= \delta_{s+t}, & [\gamma_s, \gamma_t] &= [\beta_s, \beta_t] = 0,
\end{aligned} \tag{B.2}$$

where n are integers and $\nu = 0$ in the R sector and $1/2$ in the NS sector. The ground states are defined such that they are annihilated by the positive modes of the ghosts. The zero modes of the bc ghosts, and the $\beta\gamma$ ghosts in the R sector, generate degenerate ground states. In particular, since the $\beta\gamma$ commutes, their zero modes generate an infinite tower of states. From the (anti-)commutation relations we can treat c_0 and γ_0 as a creation operators, and b and β_0 as annihilation operator. Therefore the degenerate ground states can take the form

$$\gamma_0^n c_0^l |0, k_\mu\rangle, \tag{B.3}$$

where $b_0|0,k_\mu\rangle = \beta_0|0,k_\mu\rangle = 0$. Also, $n \geq 0$ and $l \in \{0,1\}$. The ground state is infinitely degenerate. Physical states have to satisfy

$$b_0|\mathrm{phys}\rangle = \beta_0|\mathrm{phys}\rangle = 0. \tag{B.4}$$

In addition, GSO projection should remove half of the ghost states. A physical state should thus satisfy also

$$\left(1-(-1)^{F+G_{\beta\gamma}}\right)|\mathrm{phys}\rangle = 0, \tag{B.5}$$

where F is the world-sheet fermion number and $G_{\beta\gamma}$ the $\beta\gamma$ ghost number. Explicitly, in the R sector

$$G_{\beta\gamma} = \gamma_0\beta_0 + \sum_{n>0}\gamma_{-n}\beta_n + \beta_{-n}\gamma_n, \tag{B.6}$$

and similarly in the NS sector where the zero modes are absent. We are then ready to compute the partition sum. Putting in the GSO projector explicitly as in (B.1) the sum breaks down into four different terms, two in the NS sector and another two in the R sector. The trace over the world-sheet fermions implies anti-periodic boundary conditions in the world-sheet time direction. The insertion of $(-1)^F$ in the trace corresponds to flipping to periodic boundary condition. Therefore we can denote the contribution of the four terms according to the boundary conditions along $(\sigma,\ \tau)$ of the world-sheet supersymmetry current $T_F(\sigma_+) = \psi.\partial_+ X$ as (NS,NS), (NS,R), (R,NS) and (R,R). It is important that the boundary conditions of the ghosts should match those of the corresponding world-sheet current. i.e. The bc ghosts should have the same boundary conditions as the energy momentum tensor T and the $\beta\gamma$ ghosts take the boundary conditions of the world-sheet supersymmetry current. Therefore the bc ghosts always have periodic boundary conditions along both σ and τ. Given that they anti-commute, a factor of $(-1)^{G_{bc}}$ is inserted in all the traces, where G_{bc} is the bc ghosts number. For the $\beta\gamma$ ghosts their boundary conditions are as specified in the four sectors. They are commuting and so to obtain anti-periodic boundary conditions along τ we have to insert $(-1)^{G_{\beta\gamma}}$ in the trace. The partition sum in the (NS,NS) and (NS,R) sectors immediately reduce to those obtained in the gauge fixed calculation, where only the non-light-cone directions contribute. In the (R,NS) and (R,R) sector however the situation is rendered more complicated by the infinite tower of degenerate ground states due to the presence of $\beta\gamma$ zero modes. Since the trace over all states factorises for each pair of creation and annihilation operator, we can do the sum separately for the $\beta\gamma$ ghosts. In the (R,NS) sector

$$\mathrm{tr}\left((-1)^{G_{\beta\gamma}} q^{2N_{\beta\gamma}}\right) = \prod_{n=0}^{\infty} \left(\sum_m (-1)^m q^{2nm}\right). \tag{B.7}$$

The factor at $n = 0$ gives an infinite alternate series $1 - 1 + 1 \ldots$.. To regularise we take the trace

$$\mathrm{tr}\left((-x)^{G_{\beta\gamma}} q^{2N_{\beta\gamma}}\right), \tag{B.8}$$

and only take the limit $x \to +1$ at the end. The infinite series then reads

$$\sum_{n=0}^{\infty} (-x)^n = \frac{1}{1+x}. \tag{B.9}$$

This gives a factor of $1/2$ in the limit $x \to +1$. Similarly in the (R,R) sector the trace should be

$$\mathrm{tr}(q^{2N_{\beta\gamma}}) = \prod_{n=0}^{\infty} \sum_m q^{2nm}. \tag{B.10}$$

Clearly the zero modes contribution diverges. Applying the same regularisation as in (B.8) by tracing over $\mathrm{tr}((x)^{G_{\beta\gamma}} q^{2N_{\beta\gamma}})$, the zero modes give $1/(1-x)$ and diverges in the limit $x \to +1$.

Consider then the trace over the world-sheet fermions. Their zero modes satisfy the Clifford algebra. We can define (for simplicity we consider a Euclidean space)

$$\Psi_0^{\pm a} = \frac{\psi_0^{2a} \pm i\psi_0^{2a+1}}{2}, \tag{B.11}$$

where $a = 0, 1, \ldots n/2 - 1$, when there are n Neumann-Neumann or Dirichlet-Dirichlet directions. Since they satisfy anti-commutation relations

$$\left\{\Psi_0^{a+}, \Psi_0^{b-}\right\} = \delta_{ab}, \tag{B.12}$$

they can be treated as creation and annihilation operators after defining a ground state annihilated by all Ψ_0^{a-}. Consider the open strings connecting a D0 and a D8. In such cases $n = 2$. The $\Psi_0^{\pm}$ generate two degenerate ground states $|\downarrow\rangle$ and $\Psi_0^{+}|\downarrow\rangle = |\uparrow\rangle$. In the (R,NS) trace the two world-sheet fermionic ground states give a factor of $q^0 + q^0 = 2$. In the (R,R) sector the trace is

$$\mathrm{tr}\left((-1)^F q^{2N}\right). \tag{B.13}$$

The zero modes part of $(-1)^F$ can be written as $(-1)^{\Psi_0^+\Psi_0^-}$. The zero modes give $\langle\downarrow|\downarrow\rangle - \langle\uparrow|\uparrow\rangle = 1 - 1 = 0$. To *regularise* the sum as for the ghosts we can instead take the trace

$$\mathrm{tr}\left((-x)^F q^{2N}\right), \tag{B.14}$$

after which we should take the limit $x \to +1$. The zero modes contribution is then equal to $1 - x$. The total contribution of $\beta\gamma$ ghosts and world-sheet fermion zero modes are thus given by

$$\lim_{x\to+1} \frac{1-x}{1-x} = 1. \tag{B.15}$$

C. A note on monodromy in D(-1)- D7 system

The D(-1)- D7 system[6] is another example where there are eight relatively transverse dimensions between the branes and we should expect the counterpart of string creation to occur here. The configuration has codimension two, compared to Dp-D$(8-p)$ systems which have codimension one. Instead of crossing the two branes there is a two plane on which we can move the D-instanton relative to the D7. Therefore the natural counterpart of string creation in this configuration should be a monodromy in the complexified dilaton $\tau = c_0 + i2\pi e^{-\phi}$ when the D-instanton encircles the D7 brane. To exhibit this effect, we evaluate

$$\langle D-1|\Delta|D7\rangle. \tag{C.1}$$

This is analogous to what has been done for other Dp- D$(8-p)$ systems. The result is

$$\frac{A}{2\pi}\int_0^\infty \frac{dt}{t} e^{-|z|^2/(2\alpha' t)}(1 \pm 1), \tag{C.2}$$

where z is the complex coordinate of the two totally transverse dimensions. The integral should yield a harmonic function in z. Now, given that under a parity transformation that transforms $z \to e^{i\pi}z$, the result jumps from zero to one. The integral is thus

$$\langle D-1|\Delta|D7\rangle = \frac{A}{\pi}\log z. \tag{C.3}$$

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